Final Group Project Report: XYZ Manufacturing

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**Overview**

XYZ Manufacturing plans to distribute a new product, the Flugel. Our report analyzed the capacity expansion, production, and distribution for a new product at the company and determined the strategy that minimizes the total cost of meeting the expected demand over the next 10 years. Existing information we have so far has shown below (All costs are expected to rise at the rate of 3% per year):

**Retail Function：**There are 8 Retail Centers.

* Demand at each Retail Centers will grow yearly at different rates.

**Production Function:** There are 5 Plants.

* We have choice of building (and operating) new production line at plants with certain specifications.
* We may choose to shut down currently operating line or reopen previously shut-down line.
* Newly built production line incurs constructions costs and start-up costs.
* Restarting a plant only incurs re-opening costs if line has already been constructed.
* Operation costs only incurred if production occurs during year and production line cannot remain idle in given year.

**Product Resources:** Each unit of Flugel requires 2 main resources for production.

* 4.7 lbs. of alloy
  + Each plant can get maximum 60000 lbs of alloy per year.
  + Year 1 cost of alloy is 0.02 (1000s -> 20).
* 3 widget subassemblies
  + Year 1 cost of one Widget subassembly is 0.15 (1000s -> 150) but drops to 0.12 (1000s -> 120) after first 9000 Widget subassemblies have been purchased in a given year.
  + Plant can purchase as many Widgets as needed (no upper bound)

**Distribution Function:** There are 4 warehouses.

* No specific charge for storage at warehouse, but we have limit number of units stored.
* Average inventory level is average between amount of inventory t beginning of year and inventory at end of year.
* Average inventory level should be no more than 4000 units (upper bound).
* Both the flow into a warehouse and the flow out of a warehouse should not exceed an average of 1000 units per month, 12000 per year.

**Shipping Costs:**

* Shipping costs of each unit of Flugel will be generated from Plant *i* to Warehouse *j*.
* Shipping costs of each unit of Flugel from each Warehouse to each Retail Center.

**Results and Recommendations**

The results of the model can be broken down into three main categories: Costs, Shipments from Plant i Warehouse j during Year t, and Shipments from Warehouse j to Customer k in Year t. Costs are broken down into three sub-categories: construction costs/ reopening/shutdown costs, and annual operating costs.

Construction

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The model output shows that Plants 4 and 5 were built in Year 1 and thus incurred the construction costs. As they were both built in Year 1, they did not have any inflation associated with them, so it cost $900,000 for Plant 4 and $1,500,000.

Reopening/Shutdown

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Description automatically generated with medium confidenceAs both plants were built in Year 1, they also incurred the reopening costs in Year 1 as well. As it was the first year of the 10-year time span, these costs did not have the inflation associated with them as well, except for the reopening cost for Plant 4 in Year 6. For Year 1, Plant 4 had a reopening cost od $100,000 and Plant 5 was $130,000. In Year 6, Plant 4 had an additional reopening cost of $155,226.80.

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Description automatically generated with medium confidence

Plant 4 was the only plant to ever incur a shutdown cost (in this model, Plant 4 was shut down in Year 4). In Year 2, Plant 4 had a shutdown cost of $84,872.

Annual Operations

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Shipments from Plant i to Warehouse j

The shipments from Plant i to Warehouse j during Year t show how many units of finished Flugels are shipped from each plant to a respective warehouse in any given year. Theoretically, we would have shipments going from 5 different plants to four different warehouses. However, in this model, we only have shipments going to the warehouses from Plants 4 and 5.

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In this model, Plant 4 will supply finished Flugel units to Warehouses 1 and 4, but the deliveries are staggered. Plant 4 delivers to Warehouse 1 in Years 1, 6, 7, 8, 9, and 10. Plant 4 also will deliver finished Flugel units to Warehouse 4 only in Year 1.

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Seeing as Plant 4 will deliver finished Flugel units to only Warehouses 1 and 4, Plant 5 will deliver units to Warehouses 2 and 3. Plant 5 will deliver finished flugel units to Warehouse 2 for all 10 years of the time span. For Warehouse 3, Plant 5 will deliver finished Flugel units for all years expect for Year 2.

Shipments to Customer k from Warehouse j

The shipments from Warehouse j to Customer k during Year t show how many units of finished Flugels are shipped from each warehouse to a respective customer in any given year. In the model, we have shipments coming from four different warehouse going to 8 different customers.

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At this point, all four warehouses have inventories of finished flugel units. Any of these four warehouses can ship to any of the 8 customers. In the model, Warehouse 1 ships only to Customers 3 and 7. Customer 3 will receive a shipment in Years 5, 7, 8, 9, and 10. Customer 7 will receive shipments from Warehouse 1 in Years 7, 8, 9, and 10.

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Warehouse 2 ships units to almost all customers. In this case, Warehouse 2 ships to Customers 1, 2, 4, 5, 7 and 8. Most of these customers will receive yearly shipments from Warehouse 2 with the exception of Customer 8. Customers 1, 2, 4, 5, and 7 will receive shipments every year for all 10 years while Customer 8 will receive a shipment every year from Year 1 up to Year 6. Looking at Warehouse 1, they also ship to Customer 7, but only for Years 7 to Year 10, but seeing as both Warehouses 1 and 2 ship to them, their yearly demand will be covered for all 10 years.

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Warehouse 3 ships Flugel units to Customers 3, 6, and 8. Customer 3 will receive shipments in Years 1 to 6, Customer 6 will receive shipments for all 10 years of the time span, and Customer 8 will receive shipments every year from Year 6 to Year 10. With regards to Warehouses 1 and 2, they also service Customers 3 and 8 for Years 5, 7, 8, 9, and 10 and Years 1 to 6, respectively. Warehouse 3 ships to Customer 3 for the first 6 years of the time span, and to Customer 8 for Years 6 to 10, so across the three warehouses, the yearly demand for both customers will be satisfied.

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Warehouse 4 only ships to Customer 2 for Years 9 and 10. However, Warehouse 2 also ships to Customer 2 for all 10 years, so Warehouse 4 would be supplementing the last two years of their shipments to ensure that Customer 2’s demand is met.

Across all four warehouses, Warehouse 1 services Customers 3 and 7, Warehouse 2 services 1, 2, 4, 5, 7, and 8, Warehouse 3 services Customers 3, 6, and 8, and Warehouse 4 services only Customer 2. This ensures that all 8 customers are serviced by at least one warehouse and has each yearly demand satisfied.

The ultimate goal of this model is to minimize the total costs that the company will incur across construction, reopening, shutdowns, annual operating costs, shipping costs, and material costs. After using a 10-year time span, the total costs for XYZ Manufacturing is minimized to $80,751,700. Although this is a good minimization of total costs incurred, but what if the data was different?

Scenario 1

In the model, only Plants 4 and 5 are built and utilized. If we look at the capacity of each plant, they are 10000 units and 13000 units, respectively. Only Plant 5 is utilized to the fullest extent, being built in Year 1 and never shut down (operational for all 10 years) because it has a higher capacity than Plant 4. What if we were to consider a scenario where we did not build an additional plant to accommodate for Flugel production, but rather expand Plant 4’s capacity to 13000 units to match Plant 5? By doing so, we decrease our optimal minimization cost from $80,751,700 to $80,319,400, for a savings of approximately $400,000. Plant 4 would be operational for the same amount of time (6 years total, with one shutdown and one reopen), but it would save XYZ Manufacturing a good amount of money.

Scenario 2

Let’s consider one more situation: what if we kept the capacity of Plant 4 at 10000, but reduced the operating cost? Currently, the annual operating cost of Plant 4 is $280,000, but what would happen if we were to drop the operating cost to $240,000? Plant 4 would remain operational for the same duration of time (constructed and opened in Year 1, closed in Year 2, reopened in Year 6 and operational for Years 6 to 10), but the total optimal minimized costs would drop significantly. Originally it was $80, 751,700 but would drop down to $79,687,800. By simply dropping the operating costs by $40,000, we would see a total savings of over $1,00,000.

**Formulation**

**Indexed Sets:**

i = Plant i (where i = 1:5)

j = Warehouse j (where j = 1:4)

k = Retail Center k (where k = 1:8)

t= Number of the year (where t=1:10)

**Data:**

Ui = Capacity of Warehouse i

COi = Construction Costs for Plant i

Ri = Reopening Costs for Plant i

Si = Shutdown Costs for Plant i

Cij = Cost of Shipment from Plant i to Warehouse j in year t

Dk = Demand in Retail Center k

Drk = The increasing rate of Retail Center k

Sjk = Cost of Shipment from Warehouse j to Retail Center k in year t

OPi = Operating Costs for Plant i

**Decision Variables:**

Xijt = Number of units shipped from Plant i to Warehouse j during Year t（From Plant to Warehouse）

Vjkt = Number of units shipped from Warehouse j to RC k during Year t（From Warehouse to Retail Center）

IVjt = Number of units in inventory at Warehouse j at the end of Year t（Inventory at the end of year）

Pit = binary variable -> 1, if Plant i is operating during Year t; 0, if Plant i is closed during Year t

Zit = binary variable -> 1, if Plant i is constructed during Year t; 0 if otherwise

PRit = binary variable -> 1, if Plant i was reopened during Year t 0; if otherwise

PSit = binary variable -> 1, if Plant i was shut down during Year t 0; if otherwise

Over\_it =binary variable -> 1, if the unit of purchased Widget at Plant i greater than 9000; 0 otherwise.

Wit = Total Number of Widgets bought at Plant i during Year t

**Constraints-Lines Logic:**

SZit = (Sum of nodes of Z when t=1:t)

Pit <= SZit (Construction Constraint)

PiT – Pi(T – 1) = PRit – PSit for t = 2:10 (Reopen/Shutdown Constraint)

Zit <= PRit (Reopen at the same year when constructed)

Pit-Pi(t-1)+M\*PSit>=0 (Have to be closed if not operating)

**Constraints-Resources:**

Wit == ∑j Xijt \*3 (Units of Widgets used per unit of Flugel)

(Wit – 9000)\*（Over\_it-0.5）>=0 (If the units of Widget purchased greater than 9000, the node of Over\_it have to be 1; when less than or equal 9000, the node will be 0)

Wit\*4.7<= 60000 (Alloy Maximum)

Cost\_W = ∑i ∑ t(((1 - Over\_it)(Wit \* 0.15) + Over\_it(9000 \* 0.15 + (Wit-9000)\*0.12))\*(1+0.03)^(t-1)) (Cost of Widget)

Cost\_A = ∑i∑t(Wit \* 4.7 \* 0.02\*(1+0.03)^(t-1)) (Cost of Alloy)

**Constraints-Plant:**

∑jXijt <= Ui \* Pit (Supply of units from Plant i during Year t)

**Constraints-Demand:**

∑jVjkt = Dk \*(1+Dr)(t-1) for all j,k,t (Meet Demand in Year t)

**Constraints-Warehouse:** (No specific charge for storage at warehouse)

IVj1 = ∑iXij1 - ∑kVjk1 for all j, t=1 (Inventory Balance of Year 1)

IVjt-1 + ∑iXijt - ∑kVjkt = IVjt for all j, t>=2 (Inventory Balance after Year 1)

(IVjt + IVjt-1) / 2 <= 4000 for all j, all t>=2 (Inventory Maximum after Year 1)

IVj1 /2 <= 4000 for all j, t=1 （Inventory Maximum of Year 1）

∑iXijt <= 12000 (Average inventory inflow no more than 12000 yearly)

∑kVjkt <= 12000 (Average inventory outflow no more than 12000 yearly)

**Additional Constraints** (To make the formulation stronger):

PSit<=Pi(t-1) (Have to be operated before the year closed)

PRit<= (1-Pi(t-1)) (Have to be unoperated before the year reopen)

PSit+PRit <=1 (Reopen and close can not happen in the same year)

PRit<= Pit (Have to be operated at the same year of reopen)

Zit<= PRit (Have to be reopened at the same year of construction)

All variables mentioned above >=0 (non-negativity)

**Objective Function:**

Objective: Minimize Cost of Meeting Total Demand over 10 years

MIN COST =∑t ∑i (Coi \* Zit + OPi \* Pit + Ri \*PRit + Si \* PSit) \*(1+0.3)^(t-1)

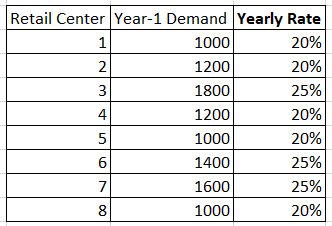
+∑t ∑i∑jCijXijt \*(1+0.3)^(t-1)

+ ∑t ∑j∑k SjkVjkt \*(1+0.3)^(t-1)

+ Cost\_W + Cost\_A

**Appendix**

* The estimated that the year-one demand for Flugels and yearly rate at each Retail center.



* The estimated year-1 costs at each plant.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | (1000's of $) |  |  |  |
| Plant | Capacity (units) | Construction Cost | Annual Operating Cost | Reopening Cost | Shutdown Cost |
| 1 | 16000 | 2000 | 420 | 190 | 170 |
| 2 | 12000 | 1600 | 380 | 150 | 120 |
| 3 | 14000 | 1800 | 460 | 160 | 130 |
| 4 | 10000 | 900 | 280 | 100 | 80 |
| 5 | 13000 | 1500 | 340 | 130 | 110 |
|  | **Yearly rate** | 3% | | | |

* The estimated year-1 cost of shipping a Flugel from Plant *i* to Warehouse *j*.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | (1000's of $) |  |  |
| Plant | Warehouse 1 | Warehouse 2 | Warehouse 3 | Warehouse 4 |
| 1 | 0.12 | 0.13 | 0.08 | 0.05 |
| 2 | 0.1 | 0.03 | 0.1 | 0.09 |
| 3 | 0.05 | 0.07 | 0.06 | 0.03 |
| 4 | 0.06 | 0.03 | 0.07 | 0.07 |
| 5 | 0.06 | 0.02 | 0.04 | 0.08 |
|  | **Yearly rate** | 3% | | |

* Resources costs per Flugel.

Table

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* The estimated year-1 unit shipping cost from each Warehouse to each Retail Center.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | (1000's of $) |  |  |  |  |  |  |  |
| Warehouse | Retail 1 | Retail 2 | Retail 3 | Retail 4 | Retail 5 | Retail 6 | Retail 7 | Retail 8 |
| 1 | 0.09 | 0.1 | 0.06 | 0.05 | 0.08 | 0.09 | 0.02 | 0.12 |
| 2 | 0.05 | 0.07 | 0.12 | 0.04 | 0.03 | 0.09 | 0.03 | 0.08 |
| 3 | 0.06 | 0.09 | 0.07 | 0.09 | 0.09 | 0.04 | 0.11 | 0.07 |
| 4 | 0.07 | 0.08 | 0.09 | 0.06 | 0.1 | 0.07 | 0.06 | 0.09 |
| **Yearly rate** | 3% | | | | | | | |